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MEASUREMENT OF THE THERMAL DIFFUSIVITY OF SOLIDS WITH AN AXISYMMETRICALLY POSITIONED SOURCE OF HEAT PULSE

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The pulse method of measurement of the thermal diffusivity of cylindrical samples is considered: an optimum version of normalization of the geometric parameters of a heat pulse, the thicknesses of a cylinder to the radius, and significance of the length of a heat pulse are discussed. The method is realized on an automated experimental setup with simultaneous recording of a thermal signal and the shape and length of a laser pulse. Nonlinear effects are eliminated by decreasing the energy density on the front surface of the sample. The setup presented allows measurement of the thermal diffusivity within a wide range of its values with an error not exceeding 5%. The obtained results of the determination of the thermal diffusivity of Al, Cu, and Fe are presented in comparison with the literature data.

The widely used pulse method (PM) of measuring thermal diffusivity [1] possesses methodical errors associated with the finite length τ and spatial distribution of the heat pulse, heat exchange with the environment, and overheating of the front surface of the sample (nonlinear effects). The error associated with the finite length can be allowed for a priori [2–6] and has the value of the correction $\sim \tau/t$ (or 0.5 $\tau/t_{1/2}$), while the error associated with the energy distribution over the cross section of the laser pulse is considerable (pulse shape) [7–9] and it is practically impossible to take it into account. Therefore, in [10, 11] it is suggested to produce rather homogeneous beams of laser radiation or to modify the method so that it becomes free of the requirement on homogeneity of the energy flux, and in [12–14], optimum dimensions of the samples relative to the parameter of the heat pulse are selected. As for heat transfer, it can be allowed for based on the shape of the obtained curve of the time dependence of temperature [6, 15–17]. One of important and universally present elements of the error in the pulse method of measurement is the nonlinear effects [18] caused by overheating of the surface, the depth of which depends on the thermal diffusivity of the sample and the length of the heat pulse ($h \sim \sqrt{a\tau}$). In [11], the value of the error of overheating of the surface is determined by the ratio of the square of the effective length of thermal diffusion $\sqrt{a\tau}$ to the square of the measurement distance $\sim \sqrt{at}$. The study of the solution of the nonlinear problem using numerical methods of calculation for cylindrical samples made of copper [19] showed that, in measurement of the thermal diffusivity, the influence of the nonlinear effect leads to an error reaching 10%.

In this work, we suggest a modification of the pulse method of measurement of thermal diffusivity which eliminates the influence of nonlinear effects by decreasing the density of the pulse energy (~0.6 J/cm^2) and increasing the sensitivity of a measurement of temperature difference, which can reach ~0.1 K; the modification involves measurement of the duration τ of the laser radiation.

The considered thermal model (Fig. 1) is a bounded cylinder of radius R with the front surface subjected to a sudden heat effect and a heat-insulated side surface. We assume the distribution of thermal energy

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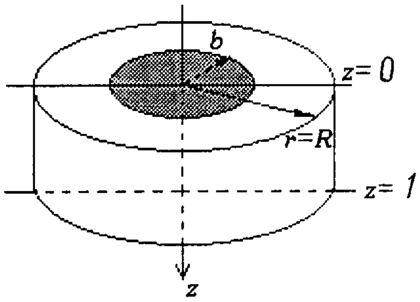

Fig. 1. Thermal model of the considered cylinder.

over the heating spot of radius b to be uniform ($b \le R$). In this case, we have axisymmetric heating, and the temperature field in the sample is described by the two-dimensional nonstationary equation of heat conduction [20, 21]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{1}$$

with initial conditions [1]

$$T(r, z, 0) = \begin{cases} \dfrac{Q}{\rho c g} & \text{when } 0 < z < g \text{ and } 0 < r < b ,\ g << l , \\ 0 & \text{when } 0 < z < g \text{ and } b < r < R , \\ 0 & \text{when } 0 < z < l \text{ and } 0 < r < R \end{cases} \tag{2}$$

and boundary conditions

$$\left.\frac{\partial T(r, z, t)}{\partial r}\right|_{r=R} = 0 , \tag{3}$$

$$\left. - \lambda \frac{\partial T(r, z, t)}{\partial z} + \alpha\, T(r, z, t)\right|_{z=0} = 0 , \tag{4}$$

$$\left. \lambda \frac{\partial T(r, z, t)}{\partial z} + \alpha\, T(r, z, t)\right|_{z=l} = 0 . \tag{5}$$

In [20], it is shown that the solution of Eq. (1) using the finite integral Hankel transform with account for heat transfer from the side and end surfaces of the bounded cylinder where the base is maintained at a constant temperature has components of the temperature fields of an unbounded plate and cylinder, and for limiting definite height-to-radius ratios of the cylinder the solution of the problem passes to one-dimensional nonstationary solutions, respectively. By virtue of the above, we determine the solution of Eq. (1) with conditions (2)–(5) in the form of the product [21, 22]

$$T(r, z, t) = T_1(r, t)\, T_2(z, t) , \tag{6}$$

where $T_1(r, t)$ and $T_2(z, t)$ are certain functions which are the solutions of the equations for an unbounded cylinder and an unbounded plate. Then the differential equation for an unbounded cylinder with initial and boundary conditions is written as

$$\frac{\partial^2 T_1(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r,t)}{\partial r} = \frac{1}{a}\frac{\partial T_1(r,t)}{\partial t}, \tag{7}$$

$$T_1(r,0) = \begin{cases} 1 & \text{when} \quad 0 < r < b, \\ 0 & \text{when} \quad b < r < R; \end{cases} \tag{8}$$

$$\left.\frac{\partial T_1(r,t)}{\partial r}\right|_{r=R} = 0 \tag{9}$$

and, correspondingly, for a plate

$$\frac{\partial^2 T_2(z,t)}{\partial z^2} = \frac{1}{a}\frac{\partial T_2(z,t)}{\partial t} \tag{10}$$

with the initial and boundary conditions

$$T_2(z,0) = \begin{cases} \dfrac{Q}{\rho c g} & \text{when} \quad 0 < z < g, \\ 0 & \text{when} \quad g < z < l, \end{cases} \tag{11}$$

$$\left.-\lambda\frac{\partial T_2(z,t)}{\partial z} + \alpha\, T_2(z,t)\right|_{z=0} = 0, \tag{12}$$

$$\left.\lambda\frac{\partial T_2(z,t)}{\partial z} + \alpha\, T_2(z,t)\right|_{z=l} = 0. \tag{13}$$

Solving Eqs. (7) and (10) with conditions (8), (9), and (11)–(13) by the method of separation of variables and series expansion, we obtain

$$T_1(r,t) = \frac{b^2}{R^2}\left(1 + 2\sum_{i=1}^{\infty}\frac{J_0\left(\frac{r\mu_i}{R}\right)J_1\left(\frac{b\mu_i}{R}\right)}{\mu_i J_0^2(\mu_i)\left(\frac{b}{R}\right)}\exp\left(-\mu_i^2\frac{at}{R^2}\right)\right), \tag{14}$$

where μ_i are the positive roots of the equation

$$J_1(\mu_i) = 0; \tag{15}$$

$$T_2(z,t) = \frac{Q}{\rho c l} \sum_{n=1}^{\infty} \frac{2\mu_n^2}{\mathrm{Bi}^2 + \mu_n^2 + 2\mathrm{Bi}} \left(\frac{\mathrm{Bi}}{\mu_n} \sin \frac{\mu_n z}{l} + \cos \frac{\mu_n z}{l} \right) \exp\left(-\frac{a\mu_n^2 t}{l^2} \right), \tag{16}$$

Here μ_n are the positive roots of the characteristic equation

$$\tan(\mu) = \frac{2\mathrm{Bi}\,\mu}{\mu^2 - \mathrm{Bi}^2}, \tag{17}$$

and the temperature field in a bounded cylinder is

$$T(r,z,t) = \frac{Q}{\rho c l} \frac{b^2}{R^2} \left(1 + 2 \sum_{i=1}^{\infty} \frac{J_0\left(\frac{r\mu_i}{R}\right) J_1\left(\frac{b\mu_i}{R}\right)}{\mu_i J_0^2(\mu_i) \left(\frac{b}{R}\right)} \exp\left(-\mu_i^2 \frac{at}{R^2} \right) \right) \times$$

$$\times \sum_{n=1}^{\infty} \frac{2\mu_n^2}{\mathrm{Bi}^2 + \mu_n^2 + 2\mathrm{Bi}} \left(\frac{\mathrm{Bi}}{\mu_n} \sin \frac{\mu_n z}{l} + \cos \frac{\mu_n z}{l} \right) \exp\left(\frac{-a\mu_n^2 t}{l^2} \right). \tag{18}$$

In the case where the heat transfer from the surfaces is equal to zero ($\alpha = 0$), the solution of Eq. (1) with account for the change in the boundary conditions (4) and (5) takes the form

$$T(r,z,t) = \frac{Q}{\rho c l} \frac{b^2}{R^2} \left(1 + 2 \sum_{i=1}^{\infty} \frac{J_0\left(\frac{r\mu_i}{R}\right) J_1\left(\frac{b\mu_i}{R}\right)}{\mu_i J_0^2(\mu_i) \left(\frac{b}{R}\right)} \exp\left(-\mu_i^2 \frac{at}{R^2} \right) \right) \left(1 + 2 \sum_{n=1}^{\infty} \cos\left(\frac{n\pi z}{l} \right) \exp\left(\frac{-n^2\pi^2 at}{l^2} \right) \right). \tag{19}$$

Considering the temperature field on the back surface of the cylinder $z = l$, we write expressions (18) and (19) in the dimensionless coordinates

$$\theta = \left(1 + 2 \sum_{i=1}^{\infty} \frac{J_0(k\mu_i)\, J_1(d\mu_i)}{d\mu_i J_0^2(\mu_i)} \exp\left(-\mu_i^2 s^2 \,\mathrm{Fo} \right) \right) \sum_{n=1}^{\infty} \frac{2\mu_n^2}{\mathrm{Bi}^2 + \mu_n^2 + 2\mathrm{Bi}} \left(\frac{\mathrm{Bi}}{\mu_n} \sin \mu_n + \cos \mu_n \right) \exp\left(-\mu_n^2 \,\mathrm{Fo} \right), \tag{20}$$

$$\theta = \left(1 + 2 \sum_{i=1}^{\infty} \frac{J_0(k\mu_i)\, J_1(d\mu_i)}{d\mu_i J_0^2(\mu_i)} \exp\left(-\mu_i^2 s^2 \,\mathrm{Fo} \right) \right) \left(1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp(-n^2\pi^2 \,\mathrm{Fo}) \right). \tag{21}$$

Figure 2a presents the dependences of the relative temperature on the back surface of the cylinder on the Fourier number for different geometric dimensions of the bounded cylinder, parameters of the heat pulse, and radial coordinates normalized to the cylinder radius in the case where Bi = 0, and Fig. 2b shows these dependences for different values of Bi. An analysis shows that the influence of the radial flux is absent and corresponds to that in [1] (Fig. 2a) at any k and s if $d = 1.0$, and also for any values of d if $s \geq 2$ [12]. However, for other values of d and s, for example, $d = 0.5$, $s = 1$, and $k = 0$, the influence of the radial heat flux is substantial and the methodical error in determination of the Fourier number amounts to more than

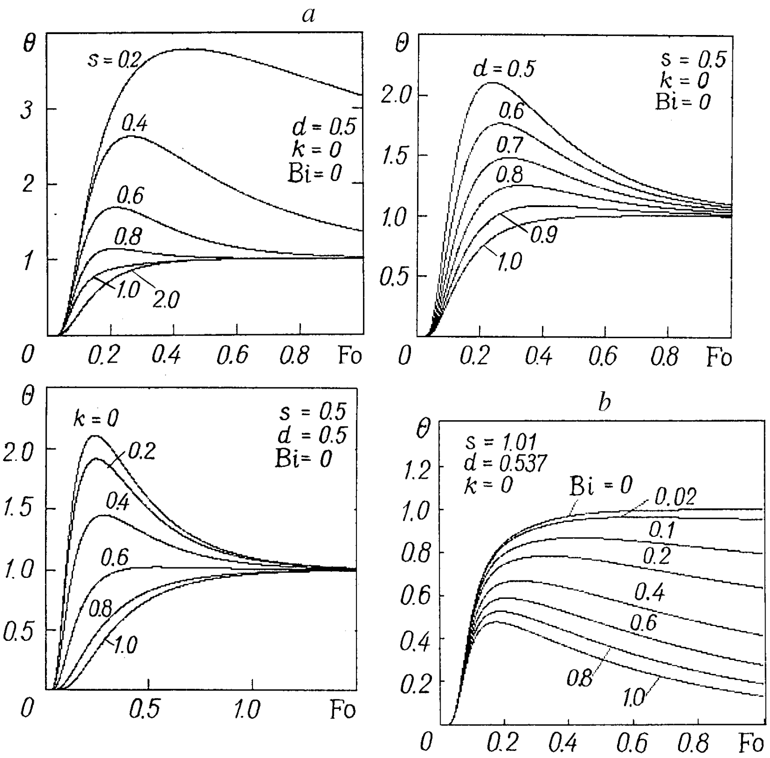

Fig. 2. Relative temperature vs. Fourier numbers ($\theta = f(\mathrm{Fo})$): a) for different thicknesses, dimensions of the heat pulse, and radial coordinates normalized to the cylinder radius (Bi = 0); b) for different values of Bi.

40% relative to the cases presented above. This influence of the axial and radial heat fluxes is more substantial when an orthotropic cylinder is considered [23].

As for the error caused by heat transfer, in our experiments (Fig. 2b) for $\mathrm{Bi} < 0.02$ [6] it does not exceed 1.5%.

A characteristic feature of the pulse method of measurement is the presence of the finite length τ of a heat pulse when the pulse shape can be represented in the form of the difference of two step functions $H(t)$ and $H(t-\tau)$ [5, 7, 24]:

$$\Psi(t) = H(t) - H(t-\tau)\,. \tag{22}$$

Integrating expression (19), which is reduced to dimensionless form, with respect to time

$$\theta = \frac{1}{\tau}\int_0^t \left(1 + 2\sum_{i=1}^{\infty} \frac{J_0\left(\frac{r\mu_i}{R}\right) J_1\left(\frac{b\mu_i}{R}\right)}{\mu_i J_0^2(\mu_i)\left(\frac{b}{R}\right)} \exp\left(-\mu_i^2 \frac{a(t-t')}{R^2}\right)\right)\left(1 + 2\sum_{n=1}^{\infty}(-1)^n \exp\left(\frac{-n^2\pi^2 a(t-t')}{l^2}\right)\right)\Psi(t)\,dt'\,, \tag{23}$$

we obtain the distribution of the temperature on the back surface of the sample:

for $\mathrm{Fo} \le \mathrm{Fo}_\tau$

$$\theta = \frac{\mathrm{Fo}}{\mathrm{Fo}_\tau} + 2\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2\pi^2\mathrm{Fo}_\tau}\left(1 - \exp\left(-n^2\pi^2\mathrm{Fo}\right)\right) +$$

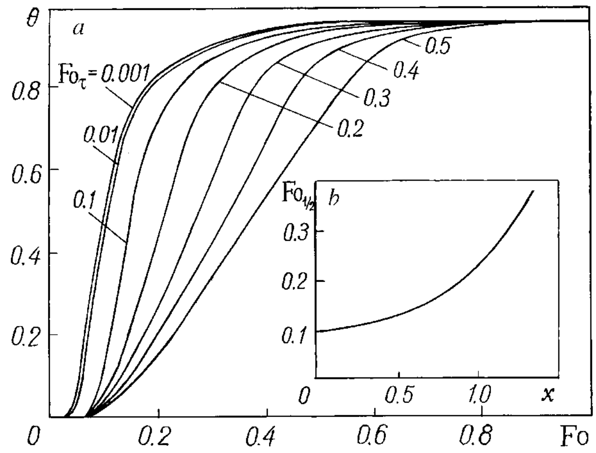

Fig. 3. Influence of the pulse length (in relative coordinates) on the change in the temperature of the back surface of the cylinder with time (a); the value of the $Fo_{1/2}$ number (b) (s = 1.01, d = 0.537, k = 0, and Bi = 0).

$$+2\sum_{i=1}^{\infty}\frac{1}{ds^2\mu_i^3\,Fo_\tau}\frac{J_0(k\mu_i)\,J_1(d\mu_i)}{J_0^2(\mu_i)}\left(1-\exp\left(-\mu_i^2s^2\,Fo\right)\right)+$$

$$+4\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\mu_i^2s^2Fo_\tau+n^2\pi^2Fo_\tau}\right)\frac{J_0(k\mu_i)\,J_1(d\mu_i)}{\mu_i\,J_0^2(\mu_i)\,d}\left(1-\exp\left(-(\mu_i^2s^2+n^2\pi^2)\,Fo\right)\right);$$

for $Fo \geq Fo_\tau$

$$\theta=1+2\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^2\pi^2Fo_\tau}\left(\exp\left(-n^2\pi^2\,(Fo-Fo_\tau)\right)-\exp\left(-n^2\pi^2Fo\right)\right)+$$

$$+2\sum_{i=1}^{\infty}\frac{1}{ds^2\mu_i^3Fo_\tau}\frac{J_0(k\mu_i)\,J_1(d\mu_i)}{J_0^2(\mu_i)}\left(\exp\left(-\mu_i^2s^2\,(Fo-Fo_\tau)\right)-\exp\left(-\mu_is^2\,Fo\right)\right)+$$

$$+4\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\mu_i^2s^2Fo_\tau+n^2\pi^2Fo_\tau}\right)\frac{J_0(k\mu_i)\,J_1(d\mu_i)}{\mu_i\,J_0^2(\mu_i)\,d}\times$$

$$\times\left(\exp\left(-(Fo-Fo_\tau)\,(\mu_i^2s^2+n^2\pi^2)\right)-\exp\left(-(\mu_i^2s^2+n^2\pi^2)\,Fo\right)\right). \tag{24}$$

The value of the thermal diffusivity of the material is determined as

$$a=Fo_{1/2}\,\frac{l^2}{t_{1/2}}\,. \tag{25}$$

The calculations made on the basis of (23) and (25) for our samples show that the disregarded pulse length equal to 20 msec leads to an error in determination of the thermal diffusivity which exceeds 10%. In

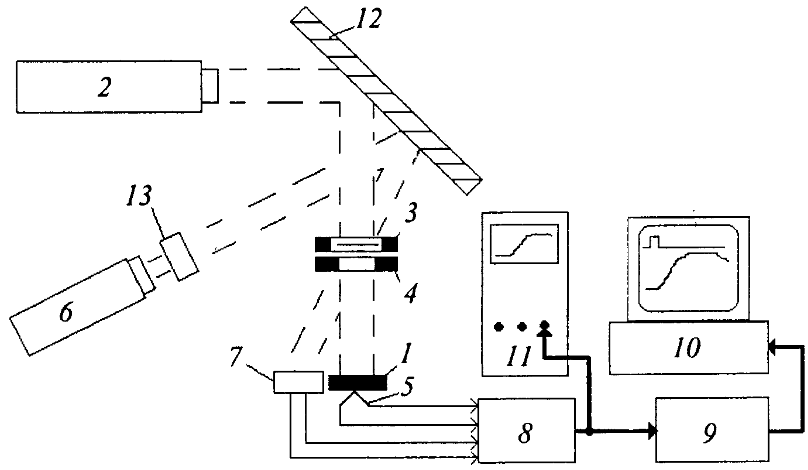

Fig. 4. Schematic of the experimental setup: 1) sample; 2) ILGN-704 laser; 3) mechanical shutter; 4) diaphragm; 5) thermocouple; 6) He–Ne laser; 7) photodiode; 8) block of amplifiers; 9) analog-to-digital converter; 10) IBM PC; 11) oscilloscope; 12) mirror plate; 13) lens.

an effort to eliminate it, on the basis of an analysis of expression (24) (Fig. 3a) for different values of Fo_τ we determined dependences of the form $Fo_{1/2} = f(x)$ which allow determination of the corrected value of $Fo_{1/2}$ corresponding to a finite pulse length. Thus, in the case $s = 1.01$, $d = 0.537$, and $k = 0$, this nonlinear dependence (Fig. 3b), with an error not exceeding 0.5%, is described by an expression of the form

$$Fo_{1/2}(x) = 0.046x^4 + 0.036x^2 + 0.05x + 0.0974\,, \qquad (26)$$

which makes it possible, in contrast to [4, 5], where the correction for length is linear, to avoid a large error in determination of the thermal diffusivity, which amounts to more than 15% even when $x = 1$.

Figure 4 shows a modernized experimental setup which differs from that presented in [6, 14]. The studied sample 1, whose front surface is coated with a thin layer of soot, is connected at the edge to the holder with the aid of a heat-insulating paste (liquid glass and Al_2O_3). The heat pulse fed to the sample 1 is produced by an ILGN-704 gas laser 2 of wavelength 10.6 μm and power ~25 W with the mechanical shutter 3, which is capable of regulating the heat-pulse length from 4 to 65 msec. In our experiments, the temperature difference, as a rule, does not exceed 5 K and 0.5 K on the front and back ends of the cylinder, respectively. The uniformity of the intensity distribution over the heating spot is achieved by distinguishing the central part of the beam with the aid of the diaphragm 4; in this case, the ratio of the spot diameter to the diameter of the beam of the laser 2 is always < 0.5. The uniformity is monitored by thermopaper. The thermal signal (Fig. 5) is recorded using the Chromel–Coppel thermocouple 5 with 0.05-mm-thick electrodes (the thermocouple is welded to the back surface of the sample along the cylinder axis ($k = 0$)), and the beginning of the action and the length of the heat pulse are rewided by the auxiliary He–Ne-laser 6 with a wavelength of 0.632 μm and an FD-24 K photodiode 7. Signals from the thermocouple and photodiode (1 and 2 in Fig. 5) are amplified in the block 8 assembled on the basis of 140UD17 operating amplifiers according to the scheme of measuring amplifiers with suppression of an in-phase component [14]. From the outlet of the block 8, they simultaneously get to the analog-to-digital converter 9 with subsequent processing by the computer 10 and to the inlet of an S8-17 storage oscilloscope 11 whose driven sweep is activated by the leading edge of the pulse of the signal from the photodiode 7. The signals are processed and the value of the thermal diffusivity is calculated by an IBM PC 10. In this case, the time resolution is 120 μsec. The thicknesses of the samples were measured on an IZV-21 vertical length meter with an accuracy of 0.001 mm. The diameters of

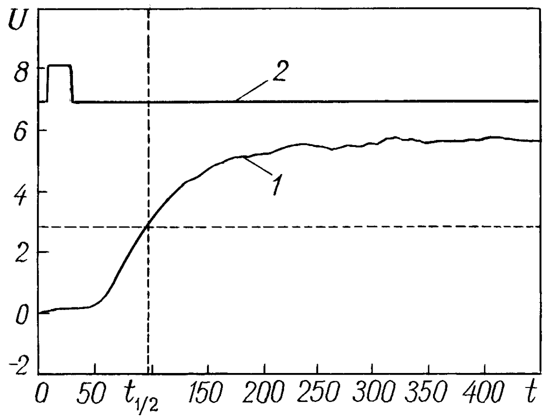

Fig. 5. Experimental dependence of the thermal signal on the time recorded on the iron sample (l = 3.950 mm, R = 3.909 mm, b = 2.100 mm, and k = 0). U, μV; t, msec.

TABLE 1. Coefficients of Thermal Conductivity ($a \cdot 10^6$ m^2/sec) from the Data of Experiments

Metal	1	2	3	4
Iron	18.8	18	–	25
Aluminum	94.5	94	93.8	98
Copper	119.8	–	117	131

Note: 1) our data; 2) [1]; 3) [25]; 4) [26]

the sample and the diaphragm and the coordinate of thermocouple location were determined using a BMI-1Ts instrumentation microscope with an accuracy of 0.001 mm.

The method was tested at a temperature of 300 K on samples made of Armco iron, copper (99.99), and aluminum (99.99). Results of the experiments are tabulated. The results obtained have good reproducibility and agree with the existing literature data [1, 25, 26]. The measurement error for thermal diffusivity does not exceed 5% and consists of the error in determination of $Fo_{1/2}$ (no higher than 3%) (a), the measurement error introduced by the inaccuracy of determination of the geometric dimensions of the samples (no higher than 0.2%) (b), and the error caused by inaccurate determination of the delay time of the temperature signal (no higher than 1.5%) (c).

Thus, the setup described allows one to measure the value of a within a wide range of thermal diffusivity, it is distinguished by the capability for recording simultaneously of a thermal signal, the shape and length of a heat pulse, and small temperature differences with the influence of high overheatings of the sample being excluded.

NOTATION

a, thermal diffusivity; τ, heat-pulse length; t, time; $t_{1/2}$, time in which the temperature signal reaches half the maximum value and which is reckoned from the leading edge of the heat pulse; h, depth of overheating of the surface; R, cylinder radius; b, radius of the heating spot; l, sample thickness; r, z, cylindrical coordinates; ρ, density; c, heat capacity; α, heat-transfer coefficient; λ, thermal conductivity; Q, energy density; g, thin layer on the front surface of the cylinder $z = 0$ ($g << l$) of radius b where the heat flux with the energy density Q is absorbed instantly; T, temperature; $J_m(x)$, Bessel function of the first kind of mth order; $Bi = \alpha l/\lambda$, dimensionless Biot number; U, thermo electromotive force; $\theta = T(r, l, t)\rho c l R^2/(Qb^2)$, dimensionless temperature; $Fo = at/l^2$, Fourier number; $Fo_{1/2}$, Fourier number corresponding to half the maximum value of the relative temperature; $Fo_\tau = a\tau/l^2$, Fourier number corresponding to the heat-pulse length; $s = l/R$, $d = b/R$, $k = r/R$, dimensionless coordinates; $H(t)$ and $H(t-\tau)$, unit step functions; $x = \tau/t_{1/2}$, ratio of the pulse length to the time in which the temperature signal reaches half the maximum value.

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